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ON THE CRITICAL DROPLET SIZE IN AN ACOUSTIC FIELD

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For Reynolds numbers Re << 1 an analytical relation is obtained for the dependence of the critical size of a liquid droplet on the amplitude and frequency of a sound field, surface tension, dynamic viscosity of gas with allowance for prehistory of motion and reattachment of mass.

Rather small particles suspended in a gaseous medium are entrained by a moving medium. The degree of entrainment, which is understood as the ratio of the amplitude of oscillations (or velocity) of a particle to the amplitude of oscillations (or velocity) of the medium, depends substantially on the physical parameters of the particle. A detailed analysis of the problem of oscillatory motion of aerosol particles in a sound field is given in a monograph by E. P. Mednikov [1].

If the amplitude of the oscillatory velocity $A\omega$ is such that Re << 1, then for a sphere of constant radius we can write the Boussinesque equation

$$\frac{4}{3}\pi\rho_{\mathrm{p}}a^3\mathbf{W}(t) = -\frac{2}{3}\pi\rho_{\mathrm{g}}a^3\mathbf{W}(t) - 6\pi\mu a\mathbf{v}(t) + \frac{1}{\sqrt{\pi\nu}}\int_0^t \frac{\mathbf{W}(\tau)\,d\tau}{\sqrt{t-\tau}}. \tag{1}$$

Assuming that $V_{gx} = D + A\omega \sin \omega t$, we represent Eq. (1) in the form

$$\mathbf{W}(t) - A\omega^2 \cos \omega t = -\frac{9\mu}{2a^2\left(\rho_{\mathrm{p}} + \frac{\rho_{\mathrm{g}}}{2}\right)}(v_{\mathrm{p}} - D - A\omega \sin \omega t) -$$

$$-\frac{9\sqrt{\mu\rho}}{2a\left(\rho_{\mathrm{p}} + \frac{\rho_{\mathrm{g}}}{2}\right)\sqrt{\pi}}\int_0^t \frac{(w_{\mathrm{p}} - A\omega^2 \cos \omega t)}{\sqrt{t-\tau}}\,d\tau. \tag{2}$$

In [1], a review of the existing methods for solving (2) is given. It is shown that an approximate solution of the problem is often limited only by allowance for viscosity, which is determined by the Stokes equation. However, failure to allow for the prehistory of motion and for reattached mass manifests itself in a comparison of theoretical and experimental studies [1, 2].

In [3] *a priori* estimates of Eq. (2) are given with allowance for the reattached mass.

However, an exact analytical solution of (2) is possible if we assume the initial velocity of the particle to be equal to zero. Actually, if $v_{\mathrm{p}}(t) = 0$, then, having introduced the substitution $v_{\mathrm{p}}(t) = \int_0^t W(\tau)d(\tau)$, we arrive at a Volterra integral equation of the second kind

$$W(t) + \alpha^2 \int_0^t W(\tau)\,d\tau + \frac{2\beta}{\sqrt{\pi}}\int_0^t \frac{W(\tau)\,d\tau}{\sqrt{t-\tau}} = f(t), \tag{3}$$

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where

$$f(\tau) = \alpha^2 D + \alpha^2 A\omega \sin \omega t + A\omega^2 \cos \omega t + \frac{2\beta}{\sqrt{\pi}} \int_0^t \frac{A\omega^2 \cos \omega\tau d\tau}{\sqrt{t-\tau}} ;$$

$$\alpha^2 = \frac{9\mu}{2a^2\left(\rho_p + \frac{\rho_g}{2}\right)} ; \quad \frac{9\sqrt{\mu\rho_0}}{2a\left(\rho_p + \frac{\rho_g}{2}\right)} = 2\beta .$$

The transform of Eq. (3) is

$$W(p)\left(1 + \frac{\alpha^2}{p} + \frac{2\beta}{\sqrt{\pi}}\right) = F(p) .$$

We find the original of $v(t)$ corresponding to the transform

$$W(p) = p \frac{F(p)}{p + 2\beta\sqrt{p} + \alpha^2} = A(p) F(p) .$$

We have

$$A(p) = \frac{p_1 + \alpha^2}{p_1 - p_2} \frac{p}{p - p_1} - \frac{p + \alpha^2}{p - p_1} \frac{p}{p - p_1} - \frac{2\beta}{p_1 - p_2}\left(\frac{p_1\sqrt{p}}{p - p_1} - \frac{p_2\sqrt{p}}{p - p_2}\right) , \tag{4}$$

where p_1 and p_2 are roots of the trinomial $(p + \alpha^2)^2 - 4\beta p$, which are equal to

$$p_1 = 2\beta^2 - \alpha^2 + 2\beta\sqrt{\beta^2 - \alpha^2} = (\beta + \sqrt{\beta^2 - \alpha^2})^2 ,$$

$$p_2 = 2\beta - \alpha^2 - 2\beta\sqrt{\beta^2 - \alpha^2} = (\beta - \sqrt{\beta^2 - \alpha^2})^2 ,$$

and, finally, after some transformations for the particle velocity we have

$$v(t) = \left[\frac{1}{2\sqrt{\beta^2 - \alpha^2}}\left(\frac{e^{p_1 t}\,\mathrm{erf}\,(\sqrt{p_1 t})}{\beta + \sqrt{\beta^2 - \alpha^2}} - \frac{e^{p_2 t}\,\mathrm{erf}\,(\sqrt{p_2 t})}{\beta - \sqrt{\beta^2 - \alpha^2}}\right) + \frac{1}{\alpha^2}\right] f(t) . \tag{5}$$

We analyze the expression

$$\left(\frac{e^{p_1 t}\,\mathrm{erf}\,(\sqrt{p_1 t})}{\sqrt{p_1}} - \frac{e^{p_2 t}\,\mathrm{erf}\,(\sqrt{p_2 t})}{\sqrt{p_2}}\right) \frac{1}{2\sqrt{\beta^2 - \alpha^2}} \equiv \kappa .$$

It is easy to see that $1/\alpha^2 >> \kappa$ for $t = 0.01$ sec. Therefore,

$$v(t) = D + A\omega \sin \omega t + \frac{A\omega^2 \cos \omega t}{\alpha^2} + \frac{2\beta}{\sqrt{\pi}\,\alpha^2} \int_0^t \frac{A\omega^2 \cos \omega\tau d\tau}{\sqrt{t-\tau}} . \tag{6}$$

By substitution, we can reduce the integral in Eq. (6) to the expression

$$\frac{\omega}{\sqrt{\pi}} \int_0^t \frac{\cos \omega\,(t - u)\,du}{\sqrt{u}} = \sqrt{2\omega}\left(\cos \omega t c\left(\sqrt{\left(\frac{2\omega t}{\pi}\right)}\right)\right) + \sin \omega t s\left(\sqrt{\left(\frac{2\omega t}{\pi}\right)}\right) ,$$

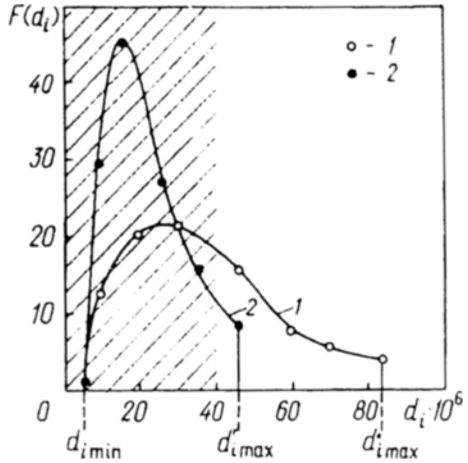

Fig. 1. Differential curves of the distribution of droplets in a jet of sprayed liquid (d_i, m): without (1) and with (2) oscillations being imposed.

where $c(x)$ and $s(x)$ are Fresnel integrals. Replacing $c(x)$ and $s(x)$ by their maximum values, we obtain the droplet relative velocity

$$v_{\mathrm{rel}}(t) = \frac{A\omega^2}{\alpha^2}\cos\omega t + \frac{2\beta}{\alpha^2}A\omega\sqrt{2\omega}\,(\cos\omega t + \sin\omega t)\,. \tag{7}$$

We find the limiting radii of droplets which are stable to oscillations. A droplet preserves its shape due to internal molecular forces. The internal pressure in it is $p = 2\sigma/r$. Due to the resistance force, the droplet is destroyed in relative motion in a gaseous medium. Assuming that the resistance force affects only half of the sphere and its numerical value is determined by the maximum values of overload, we find from the condition of equality of the forces affecting the droplet that

$$\frac{2F_{\max}}{s} + \frac{2\sigma}{r_{\mathrm{cr}}} = 0\,, \quad F_{\max} = m_{\mathrm{p}}\left(\frac{A\omega^3}{\alpha^2} - \frac{2\beta}{\alpha^2}A\omega^2\sqrt{2\omega}\right). \tag{8}$$

It is easily seen that $A\omega^2/\alpha^2 >> (2\beta/\alpha^2)A\omega^2\sqrt{2\omega}$ for $\rho_{\mathrm{p}} >> \rho_{\mathrm{g}}$, and, consequently, the critical radius of the droplet is

$$r_{\mathrm{cr}} = \sqrt[4]{\left(\frac{27}{8} - \frac{\mu\sigma}{A\omega^3\rho_{\mathrm{p}}^2}\right)}\,. \tag{9}$$

A number of assumptions made in the derivation of (9) do not hinder good agreement between the experimental values of the mean diameter of droplets of sprayed liquid and the characteristics of the acoustic field.

Figure 1 presents an experimentally obtained qualitative picture of the acoustic-field effect on the dispertivity of liquid spraying. A strict inverse dependence of the critical maximum droplet diameter on the frequency of gaseous-medium oscillations is observed.

The critical droplet radius is calculated by the formula

$$r_{\mathrm{cr}} = \sqrt{\left(\frac{27}{8}\,\frac{\mu\sigma}{\omega^2\rho_{\mathrm{p}}^2}\,\frac{\sqrt{\rho_{\mathrm{g}}c}}{\sqrt{2I}}\right)}\,, \quad \text{since} \quad A\omega = \sqrt{\left(\frac{2I}{\rho_{\mathrm{g}}c}\right)}\,.$$

Thus, at a frequency of 12 kHz and a sound pressure of 154 dB (the intensity $I = 0.1$), the critical droplet diameter is 40 μm (according to the experiment, 55 μm). In Fig. 1, the calculated region of the distribution of the disperse phase is hatched.

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NOTATION

v, velocity, m/sec; W, acceleration, m/sec^2; A, amplitude, m; ω, angular frequency, sec^{-1}; μ, dynamic viscosity, Pa·sec; ρ, density, kg/m^3; σ, surface tension, N/m^2; I, sound intensity, dB; m, mass, kg; τ, current time, sec; a, droplet radius, m; v_{gx}, axial velocity of gas; D, constant component of gas velocity.

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